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ON ISOMORPHIC CLASSICAL DIFFEOMORPHISM GROUPS. II

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Abstract

We prove that the geometric structures defined by a volume form or a symplectic form (with a mild additional condition) on smooth manifolds are determined by their automorphism groups. This is a contribution to the Erlangen Program of Klein.

1. Introduction

The goal of this paper is to prove that the geometric structures defined by a volume form or a symplectic form (with a mild additional hypothesis) on smooth manifolds are determined by their automorphism groups. This is a contribution to the Erlangen Program of Klein [7].

We draw heavily on Filipkiewicz's paper [6] in which he proves that the differentiable C^r structures on manifolds are determined by their automorphism groups, i.e., the group $\mathrm{Diff}^r(M)$ of C^r diffeomorphisms of M for $1 \leq r \leq \infty$, Filipkiewicz in turn draws heavily on Whittaker's paper [12], where the above result is established for $r = 0$, i.e., for homeomorphisms. In order to handle the differentiable case, Filipkiewicz had to avoid the infinite patching methods used by Whittaker in [12]: he developed the necessary new machinery essentially in §2 of his paper.

Unfortunately, his new arguments fail in the volume preserving case and, a fortiori, in the symplectic case. Indeed, a key lemma (his Lemma 2.1) seeks, $\forall a \in (0,1]$, a diffeomorphism (which is a product of commutators) which has the property to take a ball of radius 1 into a ball of radius a. Obviously, this cannot happen in our case. Besides, all the results of §2 are based on this lemma and on Epstein's theorem [5]. However, it is clear that Epstein's axioms are not satisfied in our cases.

The starting point of the investigations reported here was the observation that, nevertheless, some of his conclusions (for instance the conclusion of

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