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ON LIE'S APPROACH TO THE STUDY OF TRANSLATION MANIFOLDS

JOHN B. LITTLE

1. Introduction

In this paper, we will study a question originally posed by Sophus Lie in [4]. After Lie completed his work on double translation surfaces, he began to consider several different kinds of generalizations of this special class of surfaces. To understand the particular types of manifolds he considered, it should be kept in mind that Lie interpreted his theorem showing that every nondevelopable double translation surface is a piece of the theta-divisor in the Jacobian of an algebraic curve of genus three as a characterization of the abelian integrals on the curve by the functional equations they satisfy as a result of Abel's theorem. Therefore, in addition to studying the higher-dimensional analogs of double translation surfaces, Lie also proposed the problem of determining if the abelian integrals on algebraic curves of a given genus may be characterized by other, more general, functional equations.

As a first step in a program left incomplete at his death, Lie undertook the study of analytic hypersurfaces $S \subset \mathbb{C}^4$ with two different parametrizations of the form

$$x_i = \alpha_i(t_1) + A_i(t_2, t_3) = \beta_i(u_1) + B_i(u_2, u_3). \tag{1}$$

Geometrically, the existence of parametrizations of this form implies that S may be swept out in two different ways by translating a curve along a two-dimensional surface. This condition is a natural generalization of the definition of double translation three-folds, a class of manifolds which Lie had studied previously in [3] (see also [5] and [7]). In [4], Lie referred to such hypersurfaces as "Translationsmannigfaltigkeiten zweiter Art." We will call them generalized double translation manifolds instead.

The existence of two parametrizations as in (1) is *apparently* a weaker hypothesis than the assumption that S is a double translation manifold, with

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