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GLOBAL UNIQUENESS IN THE DISC LIFTING PROBLEM

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1. Introduction

In a previous paper [1] the authors have considered the problem of characterizing families of analytic discs in $\mathbf{C}^n$ whose boundaries lie on a prescribed *CR* submanifold. We obtained rather precise results which describe each such disc in the ambient space as the lift of a corresponding parameter disc in the tangent space to the manifold. The construction of each lifted disc involves solving a certain system of nonlinear singular integral equations in which the parameter discs occur as parameters. For a more complete discussion of these and related matters see [1].

In the work mentioned above we were concerned only with the local problem: we showed that there exists a unique local lifted disc, corresponding to each parameter disc, when the parameters occurring in the nonlinear integral equations are sufficiently small. This local problem has an interesting global analogue: does there exist a unique lifted disc associated with each parameter disc of arbitrary size? In fact, as the parameters in the system of nonlinear singular integral equations become larger and larger, one might well expect some kind of bifurcation phenomenon to take place.

In this paper we are concerned with the global uniqueness question. In the case where the prescribed *CR* submanifold has real codimension one, we show that no such bifurcation occurs; i.e., global solutions are unique. In §3 we prove such a uniqueness theorem under rather weak assumptions on the boundary values of the disc. In §4 we give a simpler proof which requires all the discs to be continuous on $\overline{D}$. In §5 we give a counterexample to global uniqueness for a submanifold having codimension two.

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