

AN IMPROVED TOPONOGOV COMPARISON THEOREM FOR NONNEGATIVELY CURVED MANIFOLDS

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INTRODUCTION

The main result of this paper is a global comparison theorem of Toponogov type which gives improved estimates in open nonnegatively curved manifolds. Previously, estimates on such a manifold could only be made by comparison with \mathbf{R}^2 . It is apparent that further improvements can be made, which lead towards an integral formulation of the theorem.

The application presented in section 5 is intended primarily as an example of the usefulness of this comparison theorem, and not as the optimal approach to the conjecture of Cheeger and Gromoll. It does in fact seem to me that this approach must be stretched to its limit if it is intended to be used in an effort to prove the conjecture.

The following is a list of the contents of this paper:

1. PRELIMINARIES

- 1.1. Notation
- 1.2. Curvature and conjugate points
- 1.3. Rauch-Berger comparison theorem

2. THE CUT LOCUS ON A FLATTENING SURFACE OF REVOLUTION

- 2.1. Preliminaries
- 2.2. Results
- 2.3. Example

3. HINGES, TRIANGLES AND BRANCHED COVERINGS

- 3.1. Definitions and notation
- 3.2. Hinges in flattening surfaces
- 3.3. Hinges in branched coverings of flattening surfaces

Communicated by H. Samelson, July 21, 1978. This represents part of a Ph.D. dissertation written under the guidance of Professor Jeff Cheeger at Stony Brook. It is a pleasure for the author to acknowledge the influence that Professor Cheeger has had on this work.