J. DIFFERENTIAL GEOMETRY
15 (1980) 135–159

NUMERICAL EXPERIMENTS CONCERNING THE EIGENVALUES OF THE LAPLACIAN ON A ZOLL SURFACE

GERALD R. CHACHERE

0. Introduction

A Zoll surface is a surface of revolution in $\mathfrak{R}^3$ diffeomorphic to S, the unit 2-sphere, with the property that all geodesics are closed with period 2π. S is, for example, a Zoll surface. O. Zoll [7] first proved the existence of a Zoll surface not isometric to S.

Eigenvalues of Δ, the Laplacian defined on functions whose domain is S, are $l(l + 1)$ with multiplicities $2l + 1$ ($l = 0, 1, 2, \cdots$). For Zoll surfaces other than S, each multiple eigenvalue $l(l + 1)$ splits into a "cluster" of eigenvalues near $l(l + 1)$. Specifically, a result of Alan Weinstein [5] states that there is a number $M > 0$ independent of l such that the eigenvalues of the l-th cluster are contained in the interval $[l(l + 1) - M, l(l + 1) + M]$.

In this work the following question is addressed: for a Zoll surface what is the structure of the l-th cluster for large l? Numerical experiments were made where by the eigenvalues of the first 15 or 20 clusters of selected Zoll surfaces were approximated. These computations led to two conjectures. Conjecture 1: the arithmetic mean of the eigenvalues in the l-th cluster approaches $l(l + 1)$ as l goes to ∞. Conjecture 2: the distribution of eigenvalues in the l-th cluster approaches (in a sense to be made clear) a limiting function as l goes to ∞.

In an attempt to explain these experimental results Weinstein [6] proved a theorem. The theorem which concerns the cluster structure of Δ plus a potential function tends to corroborate the conjecture.

In §1 a method for constructing one parameter families of Riemannian manifolds isometric to Zoll surfaces is given. In §2 the Laplace-Beltrami operators corresponding to the Riemannian manifolds of §1 are written in geographic coordinates, and by separation of variables an ordinary differential operator D_ε^m is defined. In §3 cluster is defined, and information about eigenvalues of D_0^m and Δ is given. In §4 the method by which the eigenvalues

Communicated by I. M. Singer, May 18, 1978.