J. DIFFERENTIAL GEOMETRY
14 (1979) 167–177

REFLECTIVE SUBMANIFOLDS. III. CONGRUENCY OF ISOMETRIC REFLECTIVE SUBMANIFOLDS AND CORRIGENDA TO THE CLASSIFICATION OF REFLECTIVE SUBMANIFOLDS

D. S. P. LEUNG

Introduction

This note is a sequel to the author's previous notes [3], [4] where we studied geodesic submanifolds of Riemannian symmetric spaces, which are the connected components of the fixed point sets of involutive isometries and are called *reflective submanifolds*. Here we shall prove that two isometric reflective submanifolds of a simply connected Riemannian symmetric space M are congruent under the full group of isometries of M. Since the second assertion in [3, Lemma 2.6] is incorrect, many of the reflective submanifolds of compact symmetric spaces listed in [4] should be replaced by their appropriate space forms. A list of the reflective submanifolds with the appropriate space form factors is given here. We also discuss in the note some facts which can be used to determine the connectivity of the fixed point sets of involutive isometries of a compact symmetric space. In our forthcoming papers, we plan to use the above results to study the various geometric significance of the reflective submanifolds. To begin with, we will classify all the real forms of Hermitian symmetric spaces. *For terminologies and notation related to reflective submanifolds and Lie groups, we follow* [4, §§1 *and* 2] *closely*.

1. Congruency of reflective submanifolds

Let $M = G/H$ be a simply connected irreducible compact symmetric space, $M^* = G^*/H$ its noncompact dual, and σ the canonical involution of M and M^*. For technical reasons, we will furthermore assume the Lie groups G and G^* to be simple and simply connected. If $\mathfrak{g} = \mathfrak{m} + \mathfrak{h}$ is the canonical decomposition of M, then $\mathfrak{g}^* = i\mathfrak{m} + \mathfrak{h} \subset \mathfrak{g}^c$, $\mathfrak{g}^c$ being the complexification of $\mathfrak{g}$ or $\mathfrak{g}^*$. Suppose ρ is an involutive isometry of M. ρ induces through adjoint

Received January 7, 1977. Supported in part by an NSF Grant.