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# COBORDISM THROUGH ONE-CODIMENSIONAL FOLIATIONS

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In a number of problems in differential geometry involving smooth structures on manifolds with boundary, a useful approach is to construct a related cobordism theory. For example, in the theory of one-codimensional foliations, the following two types of problem arise. First, given a compact connected oriented boundary $X$, one can ask if $X$ is a leaf of one-codimensional transversely oriented smooth foliation on any compact manifold which it bounds. Or, given a transversely oriented smooth one-codimensional foliation $F$ on $X$, one can ask if there exists a transversely oriented smooth one-codimensional foliation $F'$ on any compact oriented manifold bounded by $X$ which restricts to $F$ on $X$. The cobordism theory based on the first type of problem was introduced by Reinhart [1] and involves the calculation of foliated cobordism monoids $M_F^n$ in dimension $n$. The second cobordism theory is originally due to Thom and involves the calculation of foliated bordism monoids $F\Omega_1^n$. Underlying both these theories are respectively vector cobordism with associated vector cobordism groups $\Omega_V^n$ and vector bordism with associated bordism groups $V\Omega_1^n$. In [2], Reinhart introduced and completely determined the vector cobordism groups $\Omega_V^n$, showing that two closed compact oriented manifolds are vector cobordant if and only if they are cobordant in the usual oriented sense and if, in dimensions not congruent to one mod 4, they have the same Euler number or, in dimensions congruent to one mod 4, they together bound a compact oriented manifold with even Euler number. Koschorke [3] has recently determined the oriented vector bordism groups. Both of the above calculations were based on versions of the Poincaré-Hopf theorem for manifolds with boundary.

In this article, we shall compute the foliated cobordism monoids $M_F^n$ using some recent theorems of Thurston [4], [5]. With these powerful results at our disposal, the calculations turn out to be rather elementary. The results are mainly of interest in comparison with some few isolated results on the foliated bordism groups $F\Omega_1^n$. For example, using the Godbillon-Vey class (which is a bordism invariant), Thurston constructed an epimorphism of $F\Omega_1^3$ onto the real line, exhibiting an uncountable infinity of nonbordant one-codimensional foliations on the three-sphere. It turns out that in dimensions greater than two, the monoids $M_F^n$ are infinitely generated, $M_F^n$ being isomorphic to the monoid

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