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TOPOLOGY AND EINSTEIN KAEHLER METRICS

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Introduction

The main result of this paper is Theorem 4.1 which gives some interesting topological restrictions for the construction of Einstein Kaehler metrics on complex 2-manifolds. Theorem 4.1 follows quite easily from the classical index theorems and recent invariance theory calculations.

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1. Invariance theory

Let P be a map from Riemannian manifolds M to complex valued functions on M. We say that P is an invariant polynomial of order k in the derivatives of the metric g if for any $m \in M$ and smooth coordinate system normalized at origin m, $g_{ij}(m) = \delta_{ij}$, P may be expressed as a polynomial in the derivatives of g_{ij} with respect to the $\partial/\partial x_s$ such that any monomial of P contains precisely k derivatives.

These invariants have been studied extensively. However for this paper we will need only the following elementary proposition:

Proposition 1.1. *Let (M, g) be a Riemannian manifold of dimension greater than or equal to four. Denote R its curvature tensor, ρ its Ricci tensor, τ its scalar curvature, and Δ its Laplace oparator. Then $\|R\|^2, \|\rho\|^2, \tau^2, \Delta\tau$ form a basis for the invariants of order four in the derivatives of g.*

Proof. See for example [2, p. 77].

At a point m and normalized coordinate system centered at m we have $g_{ij}(m) = \delta_{ij}$ and therefore

$$\|R\|^2 = \sum_{i,j,k,l} (R_{ijkl})^2 , \qquad \|\rho\|^2 = \sum_{j,k} \Big(\sum_i R_{ijik}\Big)^2 ,$$
$$\tau^2 = \Big(\sum_{i,j} R_{ijij}\Big)^2 , \qquad \Delta\tau = \sum_{i,j,k} R_{ijij,kk} .$$

Now let Q be a map from hermitian manifolds N to complex valued func-

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