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ALMOST HERMITIAN SUBMERSIONS

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0. Introduction

In this article, we examine the differential geometric properties of almost complex Riemannian submersions between almost Hermitian manifolds (*almost Hermitian submersions*). In § 2 we define the principal objects of our study after recalling in § 1 the definitions of various almost Hermitian structures. In § 3 we prove one of our main theorems:

3.1. Theorem. *Let $f: M \to B$ be an almost Hermitian submersion with M, an almost semi-Kähler manifold. Then B is an almost semi-Kähler manifold if and only if the fibres F_y of f are minimal submanifolds of M.*

In § 4 we prove that the horizontal distribution of an almost Hermitian submersion whose total space is Kähler is completely integrable. We thereby extend a result of Ako [1, Th. 5.1, p. 502], who studied projectable tensor fields on fibre spaces with almost complex structures. We collect Ako's result as Corollary 4.1.3.

The relations between the holomorphic sectional and bisectional curvatures of the two manifolds of an almost Hermitian submersion are studied in § 5. Surprisingly, an almost Hermitian submersion whose total space is quasi-Kähler preserves holomorphic sectional curvature on horizonal vectors and *decreases* the holomorphic bisectional curvature on pairs of horizontal vectors in contrast to the well-known increasing effect of Riemannian submersions on the Riemannian sectional curvature. § 6 extends the results on Betti numbers in [19] to inequalities on the numbers $b_{1,0}$ and $b_{0,1}$ of linearly independent harmonic 1-forms of bidegrees (1, 0) and (0, 1) respectively. In particular, we find

6.5. Theorem. *Let M and B be compact almost semi-Kähler manifolds. If there exists an almost Hermitian submersion $f: M \to B$, then*

$$b_{1,0}(B) \leq b_{1,0}(M) , \qquad b_{0,1}(B) \leq b_{0,1}(M) .$$

Finally in § 7 we show that almost quaternionic submersions between q-quasi-Kähler manifolds are locally product mappings.

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