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WINDING NUMBERS AND THE SOLVABILITY CONDITION (Ψ)

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Introduction

In [3] R. Moyer has proposed a new formulation of the solvability conditions (P) and (Ψ) for pseudodifferential equations of principal type (see [4], [5]). Moyer relates these conditions to an index or winding number whose meaning is very clear and natural, when the principal symbol of the operator under study has the property that the Poisson bracket $\{p, \bar{p}\}$ does not vanish at any point where p itself does ($\bar{p}$ is the complex conjugate of p). In this case, Property (Ψ) simply says that $(1/i)\{p, \bar{p}\}$ should be >0 at any such point.

In the present paper we show that Property (Ψ) for an arbitrary symbol p without critical points is equivalent to the fact that p is the limit, in the local C^1 topology, of symbols having the above property[1]. Such a result points to a new definition of (Ψ). In our view the new definition has a two-fold advantage: first, it shows that the principal symbols of the pseudodifferential equations of principal type, whose solvability has been established so far (and which do not yet include all those satisfying (Ψ)), are limits of symbols of the kind alluded to above, and whose solvability has been well-understood (cf. [2]); secondly and perhaps most importantly, it is totally independent of the concept of *bicharacteristic*, and thus lends itself perfectly to generalization to arbitrary symbols with an arbitrary multiplicity of the characteristics or even degenerating on certain subsets. This of course leads to a new general conjecture on the necessity of (Ψ), redefined as indicated, for local solvability of any linear differential or pseudodifferential equation (see § 3).

1. Noninvolutive functions and their signatures

We shall first explain the notation used throughout the article. We shall deal with an even-dimensional Euclidean space $\boldsymbol{R}^{2n} = \boldsymbol{C}^n$, where the variable is denoted by (x, y), $x = (x_1, \cdots, x_n)$, $y = (y_1, \cdots, y_n)$, or by $z = x + \sqrt{-1}y = (z_1, \cdots, z_n)$. In application to partial differential equations, $\boldsymbol{R}^{2n}$ serves as

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[1] That a fact of this kind might be true was suggested approximately ten years ago to L. Nirenberg and the author by J. Moser.