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ALMOST CONTACT MANIFOLDS WITH KILLING STRUCTURES TENSORS. II

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1. Introduction

Almost contact manifolds with Killing structure tensors were defined in [2] as nearly cosymplectic manifolds, and it was shown normal nearly cosymplectic manifolds are cosymplectic (see also [4]). In this note we study a nearly cosymplectic structure (φ, ξ, η, g) on a manifold M^{2n+1} with η closed primarily from the topological viewpoint, and extend some of Gray's results for nearly Kähler manifolds [5] to this case. In particular on a compact manifold satisfying some curvature condition we are able to distinguish between the cosymplectic and non-cosymplectic cases. In addition, we show that if ξ is regular, M^{2n+1} is a principal circle bundle $S^1 \to M^{2n+1} \to K^{2n}$ over a nearly Kähler manifold K^{2n}, and moreover if M^{2n+1} has positive φ-sectional curvature, then M^{2n+1} is the product $K^{2n} \times S^1$.

2. Almost contact structures

A $(2n+1)$-dimensional C^∞ manifold M^{2n+1} is said to have an *almost contact structure* if there exist on M^{2n+1} a tensor field φ of type $(1,1)$, a vector field ξ and a 1-form η satisfying

$$\eta(\xi) = 1,\ \varphi\xi = 0,\ \eta\circ\varphi = 0,\ \varphi^2 = -I + \xi\otimes\eta\ ,$$

Moreover, there exists for such a structure a Riemannian metric g such that

$$\eta(X) = g(\xi, X)\ , \qquad g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y)\ ,$$

where X and Y are vector fields on M^{2n+1} (see e.g., [14]). Now define on $M^{2n+1} \times R$ an almost complex structure J by

$$J\left(X, f\frac{d}{dt}\right) = \left(\varphi X - f\xi,\ \eta(X)\frac{d}{dt}\right)\ ,$$

where f is a C^∞ function on $M^{2n+1} \times R$, [15]. If this almost complex structure is integrable, we say that the almost contact structure is *normal*; the condition for normality in terms of φ, ξ and η is $[\varphi, \varphi] + \xi\otimes d\eta = 0$, where $[\varphi, \varphi]$ is the

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