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THE REFLECTION PRINCIPLE FOR MINIMAL SUBMANIFOLDS OF RIEMANNIAN SYMMETRIC SPACES

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1. Introduction

The classical Schwartz reflection invariance of minimal surfaces states that if a minimal surface N in a Euclidean 3-space E^3 contains a straight line L, then N is locally invariant under the symmetry of E^3 with respect to L [1, p. 246]. A similar reflection principle has also been proved for a minimal surface N in a space of constant curvature which contains a geodesic L of the ambient space [7]. It is the purpose of the present note to investigate to what extent this reflection principle holds for a minimal submanifold N of a Riemannian manifold M which contains a totally geodesic manifold B of M as a hypersurface, in particular, when M is a local symmetric space.

To this end, we introduce the notion of a *reflective submanifold* of a Riemannian manifold M. An imbedded submanifold B of M is said to be *locally reflective* if there exists an involutive isometry ρ_B (i.e., $\rho_B^2 = \mathrm{id}$), called the reflection with respect to B, defined at least in an open neighborhood U of B in M such that $B \cap U$ is precisely the fixed point set of ρ_B when restricted to U. B is said to be *globally reflective* or simply *reflective* if it is complete and the isometry ρ_B is defined everywhere on M with B as its fixed point set. Using a well-known fact about the fixed point set of isometries [5, p. 61], one can conclude that every *reflective submanifold is a totally geodesic submanifold.*

We will prove the following basic facts about reflective submanifolds. *Let M be an analytic Riemannian manifold, and B a locally reflective submanifold of codimension greater than one. Then every minimal submanifold N of M, which contains B as a hypersurface, is locally invariant under ρ_B. Furthermore, if B is globally reflective and N is complete, then N is invariant under ρ_B.*

For an arbitrary Riemannian manifold, there may not exist any totally geodesic submanifold of dimension greater than one. To ensure a good supply of candidates for locally reflective snbmanifolds we will restrict our attentions to local symmetric spaces in most parts of this paper.

As with many problems related to symmetric spaces, *the problem of finding reflective submanifolds in Riemannian symmetric spaces can be reduced to*

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