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SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR

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0. Introduction

J. Simons [5] has recently proved a formula which gives the Laplacian of the square of the length of the second fundamental form, and applied the formula to the study of minimal hypersurfaces in the sphere (see also [1], [2]).

K. Nomizu and B. Smyth [4] have obtained a formula of the same type for a hypersurface immersed with constant mean curvature in a space of constant sectional curvature, and derived a new formula for the Laplacian of the square of the length of the second fundamental form, in which the sectional curvature of the hypersurface appears. Using this new formula, they determined hypersurfaces of nonnegative sectional curvature and constant mean curvature immersed in the Euclidean space or in the sphere under the additional condition that the square of the length of the second fundamental form is constant.

The purpose of the present paper is to generalize Nomizu-Smyth formulas to the case of general submanifolds and to use the formulas to study submanifolds, immersed in a space of constant curvature, whose normal bundle is locally parallelizable and mean curvature vector field is parallel in the normal bundle.

1. Preliminaries

Let there be given an n-dimensional connected submanifold M^n immersed in an m-dinensional Riemannian manifold M^m $(1 < n < m)$ with the metric tensor G, whose components are G_{ji} with respect to local coordinates $\{\xi^h\}$, (Riemannian manifolds we discuss are assumed to be differentiable and of class C^∞.) and suppose that the local expression of the submanifold M^n in M^m is

$$\xi^h = \xi^h(\eta^a) , \tag{1.1}$$

where $\{\eta^a\}$ are local coordinates in M^n. (Submanifolds we discuss are always assumed to be differentiable, of class C^∞ and connected. The indices h, i, j, k, l run over the range $\{1, \cdots, m\}$ and the indices a, b, c, d, e over the range $\{1, \cdots, n\}$. The summation convention is used with respect to these systems of indices.) Differentiate (1.1) and put

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