

## SOME HOMOLOGICAL PROPERTIES OF SPENCER'S COHOMOLOGY THEORY

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### Introduction

It has been observed by Malgrange in [4] that the theory of overdetermined systems of linear partial differential equations can be considered as a subject within the theory of differential modules. Put a little more specifically, it is clear from the results of [4] that the Spencer cohomology groups are equal to  $\text{Ext}^p(M, N)$  for appropriately chosen differential modules  $M$  and  $N$ . This statement is made precise in § 3.

What is given here is an exposition and amplification of this point of view without the restrictive hypotheses of [4]. What is accomplished by this is:

- i) Greater clarity in the ideas involved.
- ii) A more canonical and natural development of the theory.

These accomplishments are made possible by the introduction of differential module structures on jet bundles which is exploited for the first time in a systematic way here. This is the main innovation of this paper and is the missing link needed to give a fully homological account of the theory.

Concerning the greater generality of the theory presented here, an important qualification needs to be added, namely, for the differentiable case the greater generality is probably an illusion since the objects arising in the non-regular situation are apparently too little understood for us to treat them effectively.

The only contribution of the more general point of view in the differentiable case which one presently expects is the greater clarity it provides. However for the analytic, complex analytic and algebraic cases there is reason to believe that the greater generality will be quite meaningful. In this paper it is primarily the differentiable case which is treated. At appropriate points in the exposition, the minor adaptations of the theory which must be made to handle the other cases of interest are noted.

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