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# A JORDAN-HÖLDER DECOMPOSITION FOR A CERTAIN CLASS OF INFINITE DIMENSIONAL LIE ALGEBRAS

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## Introduction

**0.1** The motivation for this paper is a problem in local differential geometry, sometimes called the transitive equivalence problem. This problem will be illustrated with a typical example: Let $M$ be a Riemannian manifold. Take the curvature tensor of $M$ and its covariant derivatives of all orders and combine them by taking their products, contractions, etc. The set of all tensors obtained this way will include certain tensors of type (0, 0), which can be regarded as scalar functions on $M$. Suppose all of these functions are constant. Question: Is $M$ locally a homogeneous space? The answer due to Singer [13] is that it is.

The problem above makes sense in a much more general setting. Roughly speaking, suppose a differential structure of some sort is given on a manifold and suppose it is impossible to compute differential invariants for it which distinguish one point of the manifold from another. Is the structure, in some sense, a locally homogeneous space? For those who are familiar with Spencer's work on pseudogroup structures, this problem will be given a more precise formulation: Let $\Gamma$ be a transitive pseudogroup acting on some model space, and let $M$ be a manifold with an almost $\Gamma$ structure given on it. Problem: When does $M$ admit an underlying $\Gamma$ structure to which the given almost $\Gamma$ structure corresponds? To take a simple example: If $M$ is an almost complex manifold, when is there an underlying complex structure, here $\Gamma$ being the pseudogroup of holomorphic diffeomorphisms of $C^n$? (Cf. [4] and [15] for definitions.)

It is known that if one is given a transitive analytic pseudogroup $\Gamma$ and a normal subpseudogroup $\Gamma_0$ defined by an invariant foliation, one can define a quotient $\Gamma/\Gamma_0$ with reasonable properties; this is a theorem of Kuranishi and Rodrigues [10]. If we are given a pseudogroup and a descending chain of normal subpseudogroups, the solvability or non-solvability of the problem proposed above seems to depend only on the nature of the quotients which occur in this chain. This is rather analogous to the situation which occurs in Galois theory, where the techniques required to solve an algebraic equation

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