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FOLIATIONS AND FIBRATIONS

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1. Introduction

Let X and Y be differentiable manifolds modeled on Banach spaces, and $f : X \to Y$ a differentiable surjective map. Suppose that for each point $x \varepsilon X$ the differential $f_*(x)$ of f at x maps the tangent space $X(x)$ to X at x surjectively onto that $Y(f(x))$ to Y at $f(x)$. If the kernel of $f_*(x)$ is a direct summand of $X(x)$ (i.e., Ker $f_*(x)$ is a closed linear subspace of $X(x)$ admitting a closed supplement), then the sets $(f^{-1}(y))_{y \varepsilon Y}$ are closed differentiable submanifolds of X defining a foliation, whose leaves are the components of the manifolds $f^{-1}(y)$. We will say briefly that f *foliates* X. In particular, with each $x \varepsilon X$ there are neighborhoods $U_x, V_{f(x)}$ of $x, f(x)$ such that $f \mid U_x \to V_{f(x)}$ is a trivial fibration; see [4], [8].

The object of this paper is to impose natural conditions sufficient to insure that such a map $f : X \to Y$ is a locally trivial fibration. In finite dimensions there are several instances in which such conditions have been displayed, based on methods of Riemannian geometry; see §4 below. In particular, the work of Ehresmann [5] can be viewed as our starting point. Theorem 3C below (which was announced in [4, §5G]) includes these cases, and provides a suitable general criterion, avoiding hypotheses of finite dimensionality, separability, or of special Banach space structure of the models. Its proof therefore is necessarily different from those of the finite dimensional cases; it is based on properties of ordinary differential equations in Banach spaces, used to construct coherent liftings of paths.

In §4 we produce several special cases of Theorem 3C, extending theorems of Ehresmann [5], Hermann [6], and Rinehart [11]. Moreover, there is an important application of our theorem in the infinite dimensional case to the Teichmüller theory of Fuchsian groups. That is the object of our study [3].

2. Local lifting

In this section we adapt standard theory [2] of ordinary differential equations in Banach spaces to obtain a local path lifting property. We defer further geometric interpretation until the following section.

(A) Let E and F be Banach spaces with norms $|\ |_E$ and $|\ |_F$, and $L(F, E)$ the Banach space of all continuous linear maps of F into E, with norm denoted by $\|\ \|$. If U is an open subset of E, we consider a

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