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LAGRANGIAN TWO-SPHERES CAN BE SYMPLECTICALLY KNOTTED

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1. Introduction

In the past few years there have been several striking results about the topology of Lagrangian surfaces in symplectic four-manifolds. The general tendency of these results is that many isotopy classes of embedded surfaces do not contain Lagrangian representatives. This is called the *topological unknottedness* of Lagrangian surfaces; see [4] for a survey. The aim of this paper is to complement this picture by showing that Lagrangian surfaces can be *symplectically knotted* in infinitely many inequivalent ways. That is to say, a single isotopy class of embedded surfaces can contain infinitely many Lagrangian representatives which are non-isotopic in the Lagrangian sense. The symplectic four-manifolds for which we prove this are non-compact in a mild sense; they are interiors of compact symplectic manifolds with contact type boundary. For instance, one can take

$$M = \{z \in \mathbb{C}^3 \mid |z| < R, \quad z_1^2 + z_2^2 = z_3^{m+1} + \tfrac{1}{2}\} \tag{1.1}$$

for $m \geq 3$ and large R, with the standard symplectic form. It seems likely that the same phenomenon occurs for a large class of closed symplectic four-manifolds. At present, the best result in this direction is that for any N, there is a closed four-manifold containing N Lagrangian two-spheres which are all isotopic as smooth submanifolds but pairwise

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