J. DIFFERENTIAL GEOMETRY
51 (1999) 517-550

THE MODULI SPACE OF HYPERBOLIC CONE STRUCTURES

QING ZHOU

Introduction

A topological cone manifold is a manifold together with a link each of whose component has a cone angle attached. If each cone angle is of the form $2\pi/n$, for some integer n, the cone manifold becomes an orbifold. Unlike an orbifold, the cone manifold is not a natural concept, but it turns out to be very important in the study of geometrization of orbifolds.

In this paper, we will consider 3-dimensional geometric cone structures. The main result in this paper is an existence and uniqueness theorem for 3-dimensional hyperbolic cone structures.

Theorem A. *Let Σ be a hyperbolic link with m components in a 3-dimensional manifold X. Then the moduli space of marked hyperbolic cone structures on the pair (X, Σ) with all cone angles less than $2\pi/3$ is an m-dimensional open cube $(0, 2\pi/3)^m$, parameterized naturally by the m cone angles.*

The uniqueness part of Theorem A is an analogue of Mostow's rigidity theorem. Namely if we have two hyperbolic cone structures C_1 and C_2 with cone angles less than $2\pi/3$, then C_1 and C_2 are isometric if and only if there is a homeomorphism between (X_1, Σ_1) and (X_2, Σ_2) so that the corresponding cone angles are the same. This rigidity theorem has been improved by Kojima to hyperbolic cone manifolds with cone

Received November 11, 1997, and, in revised form, September 16, 1998. The project is partially supported by grants of NNSFC, FEYUT of SEDC and HYTEF.