

ON THE STRUCTURE OF SPACES WITH RICCI CURVATURE BOUNDED BELOW. III

JEFF CHEEGER & TOBIAS H. COLDING

0. Introduction

This paper is the third in a series devoted to the study of the structure of complete connected Riemannian manifolds, M^n , whose Ricci curvature has a definite lower bound and of the Gromov-Hausdorff limits, Y , of sequences of such manifolds.

By [6], in the noncollapsed case, off a subset of codimension ≥ 2 , such a limit space, Y , is bi-Hölder equivalent to a connected smooth Riemannian manifold (for the proof of connectedness, see Section 3 of [7].) Additionally, even in the collapsed case, there exist natural renormalized limit measures, ν , with respect to which Y is infinitesimally Euclidean almost everywhere.

In the present paper, we show that the renormalized limit measures determine a unique measure class. Moreover, with respect to any fixed such measure, ν , a limit space, Y , is a finite union of countably ν -rectifiable spaces (in the sense of [18], p. 251) and the measure, ν , is absolutely continuous with respect to the relevant Hausdorff measure. Thus, the regular part of Y is a *finite* union of spaces which, although they are not given as subsets of Euclidean space, have the properties of countably rectifiable varifolds (whose dimensions might not all be equal).

By employing rectifiability and a type $(1, 2)$ Poincaré inequality, we give a short direct argument showing that associated to the Dirichlet

Received The first author was partially supported by NSF Grant DMS 9303999 and the second author by NSF Grant DMS 9803253 and an Alfred P. Sloan Research Fellowship.