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VANISHING OF THE TOP CHERN CLASSES OF THE MODULI OF VECTOR BUNDLES

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Abstract

We prove the vanishing of the top Chern classes of the moduli of rank three stable vector bundles on a smooth Riemann surface. More precisely, the Chern class c_i for $i > 6g-5$ of the moduli spaces of rank three vector bundles of degree one and two on a genus g smooth Riemann surface all vanish. This generalizes the rank two case, conjectured by Newstead and Ramanan and proved by Gieseker.

0. Introduction

Let Y be a smooth nonsingular curve of genus $g \geq 2$ and let $M_{r,d}(Y)$ be the moduli space of stable vector bundles of rank r and degree d on Y. In case d and r are relatively prime, $M_{r,d}(Y)$ is a smooth projective variety of dimension $r^2(g-1)+1$. A classical conjecture of Newstead and Ramanan states that

$$c_i(M_{2,1}(Y)) = 0 \quad \text{for } i > 2(g-1); \tag{0.1}$$

i.e., the top $2g-1$ Chern classes vanish. The purpose of this paper is to generalize this vanishing result to higher rank cases by generalizing Gieseker's degeneration method.

In the rank 2 case, there are two proofs of (0.1) due to Gieseker [**4**] and Zagier [**16**]. Zagier's proof is combinatorial based on the precise knowledge of the cohomology ring of the moduli space: by the Grothendieck-Riemann-Roch theorem, Zagier found an expression for the total Chern class $c(M_{2,1}(Y))$ and then used Thaddeus's formula on intersection pairing to show the desired vanishing. Because the computation is extremely complicated even in the rank 2 case, it seems almost impossible to generalize this approach to higher rank cases.

A more geometric proof of the vanishing (0.1) was provided by Gieseker via induction on the genus g. Let $W \to C$ be a flat family of projective curves over a pointed smooth curve $0 \in C$ such that

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