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CHARACTERISTIC CLASSES OF FOLIATED SURFACE BUNDLES WITH AREA-PRESERVING HOLONOMY

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Abstract

Making use of the *extended* flux homomorphism defined in [**13**] on the group $\mathrm{Symp}\,\Sigma_g$ of symplectomorphisms of a closed oriented surface Σ_g of genus $g \geq 2$, we introduce new characteristic classes of foliated surface bundles with symplectic, equivalently area-preserving, total holonomy. These characteristic classes are stable with respect to g and we show that they are highly non-trivial. We also prove that the second homology of the group $\mathrm{Ham}\,\Sigma_g$ of Hamiltonian symplectomorphisms of Σ_g, equipped with the discrete topology, is very large for all $g \geq 2$.

1. Introduction

In this paper we study the homology of symplectomorphism groups of surfaces considered as discrete groups. We shall prove that certain homology groups are highly non-trivial by constructing characteristic classes of foliated surface bundles with area-preserving holonomy, and proving non-vanishing results for them.

Let Σ_g be a closed oriented surface of genus $g \geq 2$, and $\mathrm{Diff}_+\,\Sigma_g$ its group of orientation preserving selfdiffeomorphisms. We fix an area form ω on Σ_g, which, for dimension reasons, we can also think of as a symplectic form. We denote by $\mathrm{Symp}\,\Sigma_g$ the subgroup of $\mathrm{Diff}_+\,\Sigma_g$ preserving the form ω. The classifying space $\mathrm{BSymp}^\delta\,\Sigma_g$ for the group $\mathrm{Symp}\,\Sigma_g$ with the *discrete* topology is an Eilenberg-MacLane space $K(\mathrm{Symp}^\delta\,\Sigma_g, 1)$ which classifies foliated Σ_g-bundles with area-preserving total holonomy groups.

Our construction of characteristic classes proceeds as follows. Let $\mathrm{Symp}_0\,\Sigma_g$ be the identity component of $\mathrm{Symp}\,\Sigma_g$. A well-known theorem of Moser [**24**] concerning volume-preserving diffeomorphisms implies that the quotient $\mathrm{Symp}\,\Sigma_g/\,\mathrm{Symp}_0\,\Sigma_g$ can be naturally identified with the mapping class group $\mathcal{M}_g$, so that we have an extension

$$1 \longrightarrow \mathrm{Symp}_0\,\Sigma_g \longrightarrow \mathrm{Symp}\,\Sigma_g \xrightarrow{p} \mathcal{M}_g \longrightarrow 1.$$

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