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A TOPOLOGICAL MODEL FOR THE FUKAYA CATEGORIES OF PLUMBINGS

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Abstract

We prove that the algebra of singular cochains on a smooth manifold, equipped with the cup product, is equivalent to the A_∞ structure on the Lagrangian Floer cochain group associated to the zero section in the cotangent bundle. More generally, given embeddings with isomorphic normal bundles of a closed manifold B into manifolds Q_1 and Q_2, we construct a differential graded category from the singular cochains of these spaces, and prove that it is equivalent to the A_∞ category obtained by considering exact Lagrangian embeddings of Q_1 and Q_2 which intersect cleanly along B.

1. Introduction

In [**8**], Fukaya and Oh proved that counts of holomorphic discs in a cotangent bundle with boundary conditions along exact Lagrangian sections agree, in a certain degeneration, with a count of gradient trees. This has been widely expected to lead to an equivalence between the A_∞ structure defined on Floer cochains of a Lagrangian Q and the chain level cup product on the classical (Čech, simplicial, singular) models for its ordinary cohomology. Over $\mathbb{R}$, Kontsevich and Soibelman in [**11**] described an argument using the result of Fukaya and Oh and Homological perturbation theory on de Rham cohomology, which proves this result. A different proof, also over $\mathbb{R}$, is given by Fukaya, Oh, Ohta, and Ono's in their book on Lagrangian Floer cohomology [**9**, Section 33]. One corollary of this paper is that such an equivalence holds over the integers:

Theorem 1.1. *There is an A_∞ equivalence*

$$C^*(Q) \to CF^*(Q,Q). \tag{1.1}$$

Remark 1.2. In order to define $CF^*(Q,Q)$ as an honest A_∞ algebra (rather than work with partially defined algebraic structures as in [**11**]) we follow the approach used by Seidel in [**17**]. The main idea is to make choices of perturbations to ensure the genericity of all moduli

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