

ON POISSON ACTIONS OF COMPACT LIE GROUPS ON SYMPLECTIC MANIFOLDS

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Abstract

Let $G_{\mathcal{P}}$ be a compact simple Poisson-Lie group equipped with a Poisson structure \mathcal{P} , and (M, ω) be a symplectic manifold. Assume that M carries a Poisson action of $G_{\mathcal{P}}$, and there is an equivariant moment map in the sense of Lu and Weinstein which maps M to the dual Poisson-Lie group $G_{\mathcal{P}}^*$, $\mathbf{m} : M \rightarrow G_{\mathcal{P}}^*$. We prove that M always possesses another symplectic form $\tilde{\omega}$ so that the G -action preserves $\tilde{\omega}$ and there is a new moment map $\mu = e^{-1} \circ \mathbf{m} : M \rightarrow \mathcal{G}^*$. Here e is a universal (independent of M) invertible equivariant map $e : \mathcal{G}^* \rightarrow G_{\mathcal{P}}^*$. We suggest new short proofs of the convexity theorem for the Poisson-Lie moment map, the Poisson reduction theorem and the Ginzburg-Weinstein theorem on the isomorphism of \mathcal{G}^* and $G_{\mathcal{P}}^*$ as Poisson spaces.

The main goal of this paper is to compare Hamiltonian and Poisson actions of compact simple Lie groups on symplectic manifolds. We prove that one can always exchange the Poisson action to a Hamiltonian one by an appropriate change of the symplectic structure. This trick reduces many questions concerning Poisson actions to their well known counterparts from the theory of Hamiltonian G -actions. In particular, we suggest new simple proofs of the convexity theorem for the Poisson-Lie moment map [5], Poisson reduction theorem [10] and the Ginzburg-Weinstein theorem [7]. The results of this paper were announced in [2].

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