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HARMONIC MEASURES, HAUSDORFF MEASURES AND POSITIVE EIGENFUNCTIONS

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Abstract

Let M be a compact negatively curved Riemannian manifold with universal covering $\tilde{M}$, and let $\delta_0 > 0$ be the negative of the bottom of the positive spectrum of the Laplacean Δ on $\tilde{M}$. We use methods from ergodic theory to show that $\Delta + \delta_0$ admits a Green's function which decays exponentially with the distance. Moreover for almost every point $\zeta \in \partial\tilde{M}$ with respect to a suitable Borel-measure which is positive on open sets, the unique minimal positive $\Delta + \delta_0 - \epsilon$-harmonic functions on $\tilde{M}$ with pole at ζ normalized at a point $x \in \tilde{M}$ converge as $\epsilon \to 0$ uniformly on compact sets to a minimal positive $\Delta + \delta_0$-harmonic function.

1. Introduction

Let M be an n-dimensional compact manifold of negative sectional curvature, and let $\tilde{M}$ be its universal covering. For every $x \in \tilde{M}$ the harmonic measure ω^x at x is a Borel-probability measure on the ideal boundary $\partial\tilde{M}$ of $\tilde{M}$, which via the canonical identification can be viewed as a measure on the fibre $T^1_x\tilde{M}$ at x of the unit tangent bundle $T^1\tilde{M}$ of $\tilde{M}$.

Let Γ be the fundamental group of M acting as a group of isometries on $\tilde{M}$ and $T^1\tilde{M}$. For $\Psi \in \Gamma$ we then have $\omega^{\Psi x} = \omega^x \circ (d\Psi)^{-1}$, and hence the measures ω^x can be transported to measures on the fibres of the unit tangent bundle T^1M of M.

Denote by DTM (resp. $DT\tilde{M}$) the smooth fibre bundle over M (resp. $\tilde{M}$) whose fibre DTM_x at $x \in M$ (resp. $DT\tilde{M}_x$ at $x \in \tilde{M}$) equals $T^1_xM \times T^1_xM$ (resp. $T^1_x\tilde{M} \times T^1_x\tilde{M}$). We call a function β on DTM *symmetric* if β is invariant under the natural involution $(v, w) \to (w, v)$. In Section 2 of this note we show:

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