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GLOBAL EXISTENCE AND CONVERGENCE OF YAMABE FLOW

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1. Introduction

Let M^n be a closed connected manifold of dimension $n \geq 3$ and $[g_0]$ a given conformal class of metrics on M. We consider the (normalized) total scalar curvature functional $\mathbf{S}$ on $[g_0]$,

$$\mathbf{S}(g) = \frac{1}{V(g)^{(n-2)/n}} \int_M R_g \, dv_g, \qquad g \in [g_0],$$

where dv_g is the volume form of g, $V(g) = \int_M dv_g$, and R_g denotes the scalar curvature function of g. Simple computations [1], [13] show that the gradient of $\mathbf{S}$ at g is given by $((n-2)/2n)V(g)^{-1}(R_g - s_g)g$, where $s_g = V(g)^{-1} \int_M R_g \, dv_g$. The negative gradient flow of $\mathbf{S}$ is hence given by

$$\frac{\partial g}{\partial t} = \frac{n-2}{2n} V(g)^{-1}(s_g - R_g)g. \tag{1.1}$$

This flow preserves the volume, as can be easily verified. Changing time by a constant scale, we then arrive at the *Yamabe flow* introduced by Hamilton:

$$\frac{\partial g}{\partial t} = (s - R)g. \tag{1.2}$$

(The subscript g is omitted.) Along Yamabe flow, the total scalar curvature is decreased. Moreover, if the flow exists for all time and converges smoothly as $t \to \infty$, then the limit metrics have constant scalar curvature. Hence, Yamabe flow should be an efficient tool to produce metrics of constant scalar curvature in a given conformal class. Indeed, it was originally conceived to attack the Yamabe problem. Then the Yamabe

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