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COHOMOLOGY OF SCHUBERT SUBVARIETIES OF GL_n/P

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Dedicated to Professor I. M. Gelfand on his seventy-fifth birthday

Abstract

Let GL_n be the group of $n \times n$ invertible complex matrices, and P a parabolic subgroup of GL_n. In this paper we give a geometric description of the cohomology ring of a Schubert subvariety Y of GL_n/P. Our main result (Theorem 3.1) states that the coordinate ring $A(Y \cap Z)$ of the scheme-theoretic intersection of Y and the zero scheme Z of the vector field V associated to a principal regular nilpotent element $\mathfrak{n}$ of $\mathfrak{gl}_n$ is isomorphic to the cohomology algebra $H^*(Y; \mathbf{C})$ of Y. This theorem was conjectured for any reductive algebraic group G in [4], and it was proved for the Grassmannian manifolds in [2]. We were recently informed that Professor D. H. Peterson has just proved that GL_n is exactly the algebraic group G where the cohomology ring of any Schubert subvariety Y of the space G/B is isomorphic to $A(Y \cap Z)$. Here B stands for a Borel subgroup of G. It is also interesting to note that the cohomology ring of the union of two Schubert subvarieties in GL_n/P may not admit such a description. This result is due to Professor J. B. Carrell.

0. Introduction

Let X be a nonlinear complex projective variety having the following properties:

(A) there exists an algebraic vector field V with exactly one zero x_0, and

(B) there exists an algebraic $\mathbf{C}^*$-action on X

$$\lambda\colon \mathbf{C}^* \times X \to X \qquad ((t, x) \to \lambda(t) \cdot x),$$

such that $d\lambda(t) \cdot V = t^p V$ for some $p > 0$ and for all t in $\mathbf{C}^*$, where $d\lambda(t)$ is the associated tangent action of $\lambda(t)$ on vector fields.

Let Z be the zero scheme of the vector field V, and let Y be any V- and $\mathbf{C}^*$-invariant subvariety of X. It follows from property (B) that Z is a $\mathbf{C}^*$-invariant subscheme of X. Thus, the coordinate ring $A(Z)$

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