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CONFORMALLY FLAT METRICS OF CONSTANT POSITIVE SCALAR CURVATURE ON SUBDOMAINS OF THE SPHERE

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Abstract

A basic problem has been to construct complete conformally flat metrics of constant positive scalar curvature on the complement of arbitrary sets $\Lambda \subset S^n$ where S^n is an n-sphere. A necessary condition for the existence of such a metric is that the Hausdorff dimension of Λ must be less than or equal to $(n-2)/2$. Examples are known when Λ is any finite collection of points, a subsphere, and also when Λ is the limit set of certain Kleinian groups. Up until now no examples have been known where Λ is a smooth (nonspherical) submanifold of positive dimension. We prove here that there are many examples whenever Λ is a small perturbation of an equatorial subsphere. A local version of this result is also proved. These theorems rely on an analysis of certain degenerate linear elliptic operators, which is complicated by the fact that these operators have infinite dimensional null-spaces. A fairly general construction of pseudodifferential right-inverses for such operators is presented.

1. Introduction

Because of the resolution of the Yamabe problem by R. Schoen in 1984 [26] (see also [13]), it is possible to divide the conformal classes of Riemannian metrics on an arbitrary compact manifold M of dimension $n \geq 3$ into three disjoint subsets containing respectively those classes which contain a metric of constant positive, zero, or negative scalar curvature, according to the sign of the first eigenvalue of the conformal Laplacian on M. This sign is well-defined within a conformal class. On a given manifold one or more of these subsets may be empty; for example, it is known that the $\hat{A}$ genus is a topological obstruction governing the existence of metrics of positive scalar curvature when the manifold is spin. When the manifold is not assumed to be compact, these simple statements need modification. In this setting it is geometrically and analytically natural to restrict attention to complete metrics; however, a conformal class always contains both

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