J. DIFFERENTIAL GEOMETRY
32 (1990) 605–712

DENDROLOGY OF GROUPS IN LOW $\mathbb{Q}$-RANKS

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Introduction

The work of Bass and Serre [15] established the study of actions of groups on simplicial trees as a basic theme in the theory of infinite groups. From the standpoint of abstract group theory, this "simplicial dendrology" provides an efficient and elegant context for the classical theory of free products with amalgamation. From the standpoint of linear group theory, simplicial trees arise naturally as Tits buildings for rank-1 algebraic groups (such as SL_2) over discretely valued fields.

Actions of groups (by isometries) on $\mathbb{R}$-trees, or more generally on Λ-trees, where Λ is an ordered abelian group, are natural generalizations of actions on simplicial trees. (The definition of a Λ-tree is reviewed in §1 of this paper, following the point of view of [12] and [1]. The simplicial case is equivalent to the case $\Lambda = \mathbb{Z}$; see [12, p. 430].) The study of such generalized trees began with Lyndon [8] and Chiswell [3], who took the abstract group-theoretic point of view. In [8], Lyndon raised the question of which groups can act freely on $\mathbb{R}$-trees, and how the actions may be classified.

By "classifying" actions of a group Γ on $\mathbb{R}$-trees we shall mean classifying the (translation) length functions (1.16) that they define on Γ. In [1] and [4], it is shown that for a suitably nondegenerate action, the length function contains all the essential information. In this introduction we shall say that two actions of Γ on $\mathbb{R}$-trees are *equivalent* if they define the same length function. An action of a finitely generated group Γ on T defines the zero length function if and only if some point of T is fixed by all of Γ; we shall call such an action *trivial.*

The study of generalized trees from the point of view of linear group theory was initiated independently by Tits, who recognized the role of such trees as buildings for rank-1 algebraic groups over fields with indiscrete

Received July 15, 1988. The first author was partially supported by the National Science Foundation and the Sloan Foundation, and the second author by the National Science Foundation.