J. DIFFERENTIAL GEOMETRY
29 (1989) 95-103

EXAMPLES OF MANIFOLDS OF POSITIVE RICCI CURVATURE

JI-PING SHA & DA-GANG YANG

The purpose of this paper is to present some new examples of simply connected Riemannian manifolds of dimension ≥ 7 with positive Ricci curvature, which admit no metric with nonnegative sectional curvature, in both compact and noncompact cases. The noncompact examples presented here are not of finite homotopy type, i.e., they are not homotopy equivalent to the interior of a compact manifold with boundary. This settles a long-standing conjecture (cf. [1], [18]).

It has been one of the major problems in Riemannian geometry to distinguish the topological implications of nonnegative sectional curvature on a manifold M from those of positive Ricci curvature. In the case of $\dim M = 3$, there is actually no difference (all 3-manifolds with positive Ricci curvature admit metric with positive sectional curvature) by the works of R. Hamilton, R. Schoen and S. T. Yau ([14], [15]). The situation of higher dimensions is different. In the compact case, although it is generally believed that a simply connected manifold with positive Ricci curvature may not carry metric with nonnegative sectional curvature, examples were not known ([11], [18]). In the noncompact case the recent progresses are due to L. Berard Bergery, D. Gromoll and W. T. Meyer ([3], [11]). They constructed some examples of manifolds with positive Ricci curvature, which admit no metric with nonnegative sectional curvature. However, their examples are all of finite homotopy type. More recently, U. Abresch and D. Gromoll ([1]) proved that a complete manifold with positive Ricci curvature is of finite homotopy type under some diameter growth condition. This diameter growth condition turns out to be necessary as the examples in this paper show. Therefore the finiteness conjecture for complete Riemannian manifolds with positive Ricci curvature is not true in general. (Compare [6], [7], [10], [12].)

It is a beautiful theorem of M. Gromov [12] that for each positive integer n there is a constant C_n which only depends on n such that the total Betti

Received July 9, 1987. The first author was supported in part by National Science Foundation grant #DMS-8610730 (1), and the second author also by the National Science Foundation.