J. DIFFERENTIAL GEOMETRY
26 (1987) 1–31

CONSTRUCTION OF SINGULAR HOLOMORPHIC VECTOR FIELDS AND FOLIATIONS IN DIMENSION TWO

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0. Introduction

In this paper we construct holomorphic differential equations or foliations in two different situations:

Case 1. *Singularities of vector fields* (local case).

Case 2. *Ricatti foliations in* $\overline{\mathbf{C}} \times \overline{\mathbf{C}}$ (global case).

In Case 1 we consider singular vector fields defined in a neighborhood U of $0 \in \mathbf{C}^2$. Suppose that 0 is an isolated singularity of X. In this case, as is well known, the singularity can be solved by a finite number of blowing-ups (cf. [4], [5], and [12]). Let us consider for simplicity the case where X is solved by one blowing-up. After blowing-up $0 \in \mathbf{C}^2$, we obtain a complex line bundle $\tilde{\mathbf{C}}^2 \to \overline{\mathbf{C}}$, $\overline{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$, a proper projection π: $\tilde{\mathbf{C}}^2 \to \mathbf{C}^2$, and a singular holomorphic foliation $\mathscr{F}$ on $\tilde{U} = \pi^{-1}(U)$, where:

(i) $\pi^{-1}(0) = \overline{\mathbf{C}}$, the zero section of $\tilde{\mathbf{C}}^2$, and π: $\tilde{\mathbf{C}}^2 - \overline{\mathbf{C}} \to \mathbf{C}^2 - \{0\}$ is a diffeomorphism.

(ii) π sends nonsingular leaves of $\mathscr{F}$ in $\tilde{U} - \overline{\mathbf{C}}$ onto integral curves of the complex differential equation $\dot{x} = X(x)$. The singularities of $\mathscr{F}$ are in $\overline{\mathbf{C}}$ and are all simple (cf. §1.1 for the definition). Set S = set of singularities of $\mathscr{F}$.

In some cases (nondicritical cases) $\overline{\mathbf{C}}$ is invariant by $\mathscr{F}$, that is, $\overline{\mathbf{C}}$ is the union of S and a leaf of $\mathscr{F}$, $\overline{\mathbf{C}} - S$. Therefore it is possible to consider the holonomy group of the leaf $\overline{\mathbf{C}} - S$ (in some transversal section). This group is called the projective holonomy of the singularity and we denote it by $\mathscr{H}(\mathscr{F})$. In §2 of this paper we prove a slightly more general version of the following result.

Theorem 1. *Let* $G = \{g_1, \cdots, g_k\}$ *be a set of germs at* $0 \in \mathbf{C}$ *of holomorphic diffeomorphisms which leave* 0 *fixed and such that* $g_1, \cdots, g_k$ *and* $g_1 \circ \cdots \circ g_k$ *are linearizable, not necessarily in the same coordinate system. Then*

Received February 13, 1985 and, in revised form, April 4, 1986.