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METRICS OF NEGATIVE RICCI CURVATURE ON SL(n, R), $n \geqslant 3$

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1. Introduction

In his survey article J. Milnor [8] treats the problem of classifying those Lie groups which admit left invariant metrics whose sectional or Ricci curvatures have constant sign.

Sectional curvature (denoted by k) classification theorems ($k > 0$, $k = 0$, $k < 0$, $k \geqslant 0$, $k \leqslant 0$) are presented in [9], [8], [6], [3] and [2]. The analogous Ricci curvature (henceforth denoted by r) classification problem is not completely solved. The cases $r = 0$ and $r \geqslant 0$ are equivalent to $k = 0$ and $k \geqslant 0$ (see [3]). The case $r > 0$ is settled through Myers theorem [8]. To our knowledge, the classification of Lie groups which admit a left invariant metric so that all Ricci curvatures satisfy $r < 0$ or $r \leqslant 0$ is still an open problem.

It is known [2] that a connected Lie group with a nonflat left invariant metric of sectional curvature $k \leqslant 0$ is necessarily *solvable* and *non-unimodular*. Contrasting with this, J. Milnor constructs in [8] nonflat metrics of Ricci curvature $r \leqslant 0$ on the 3-dimensional *simple* group $SL(2, R)$. He then asks whether any *higher dimensional simple group* admits such a metric.

In this article, we construct left invariant metrics of strictly negative Ricci curvature on the *simple Lie group* $SL(n, \mathbf{R})$, $n \geqslant 3$. The construction of such metrics is rather computational. To the reader's benefit, we include all necessary details.

We are indebted to R. J. Miatello for bringing to our attention that, for $n \geqslant 3$, the manifolds $\Gamma \backslash SL(n, \mathbf{R})$, where Γ is a uniform, torsion free, discrete subgroup of $SL(n, \mathbf{R})$, provide a family of examples of compact riemannian manifolds of negative Ricci curvature where $SO(n)$ acts freely (compare [10], [11, p. 10]). The fact that these manifolds also admit metrics of positive scalar curvature answers another question in [10].

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