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GENERALIZED ROTATIONAL HYPERSURFACES OF CONSTANT MEAN CURVATURE IN THE EUCLIDEAN SPACES. I

WU-YI HSIANG

Introduction

In 1841, Delaunay [3] discovered a beautiful way of constructing rotational hypersurfaces of constant mean curvature in the euclidean 3-space E^3, namely, its generating curve can be obtained as the trace of a focus by rolling a given conic section on the axis. The above theorem of Delaunay was generalized to higher dimensional euclidean spaces in [9], namely, the generating curves of those $O(n-1)$-invariant hypersurfaces of constant mean curvature in E^n can again be obtained by rolling construction. However, from the viewpoint of equivariant differential geometry, a natural generalization of the rotational surfaces of E^3 should, at least, include those hypersurfaces which are invariant under an *isometric* transformation group (G, E^n) with *codimension two principal orbit type*. For example, in the case of E^4, there is the transformatiion group of type $O(2) \times O(2)$ acting on $E^2 \times E^2 = E^4$ besides the "usual" $O(3)$-action on E^4. Of course, in the final analysis, it will all depend on what kind of results such a generalization will lead to. As a preliminary indication, the study to generalized rotational hypersurfaces of $O(k) \times O(k)$-type already leads (here and the comparison paper [8]) to the discovery of a family of important new examples of constant mean curvature immersions of $(2k-1)$-spheres into E^{2k}. This result strongly suggests that the geometry of generalized rotational hypersurfaces definitely deserves a systematic investigation.

In this paper, we shall begin a systematic study of generalized rotational hypersurfaces of *constant mean curvatures* in E^n. The analytical problem of such a geometrical object can be reduced to the global solutions of certain specific ordinary differential equations. In §1, we shall recall some known results of [6,10], which will enable us to write down the reduced, ordinary differential equation for each type of generalized rotational transformation groups. One may naturally divide such transformation groups (G, E^n) into five

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