

THREE-MANIFOLDS WITH POSITIVE RICCI CURVATURE

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1. Introduction

Our goal in this paper is to prove the following result.

1.1 Main Theorem. *Let X be a compact 3-manifold which admits a Riemannian metric with strictly positive Ricci curvature. Then X also admits a metric of constant positive curvature.*

All manifolds of constant curvature have been completely classified by Wolf [6]. For positive curvature in dimension three there is a pleasant variety of examples, of which the best known are the lens spaces $L_{p,q}$. Wolf gives five