J. DIFFERENTIAL GEOMETRY
8 (1973) 259-278

ISOMETRIC IMMERSIONS OF MANIFOLDS WITH PLANE GEODESICS INTO EUCLIDEAN SPACE

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1. The main theorems

The object of this note is to prove the following

Theorem 1. *Assume that* (a) *M is an n-dimensional* ($n \geq 2$) *connected Riemannian manifold,* (b) *$f: M \to R^{n+p}$ is an isometric immersion of M into an $(n + p)$-dimensional Euclidean space R^{n+p}, $p > 0$, and* (c) *every geodesic on M is locally a plane curve, that is, if $\sigma: (\alpha, \beta) \to M$ is a geodesic on M, then for every $t \in (\alpha, \beta)$, there exists an open interval I in (α, β) containing t such that $f \circ \sigma(I)$ lies on a certain plane E_t. Then either $f(M)$ is an open subset of an n-dimensional plane or M is $\frac{1}{4}$-pinched, i.e., its sectional curvature K satisfies*

$$\tfrac{1}{4}A \leq K \leq A$$

for some positive number A.

If M is also $\frac{1}{4}$-pinched, then we have

Theorem 2. *Assume that* (a), (b), (c) *of Theorem* 1 *hold, and that M is $\frac{1}{4}$-pinched. Then M has positive constant sectional curvature, if one of the following conditions also holds:*

(1) $1 \leq p < \frac{1}{2}n + 2$,

(2) *n is prime,*

(3) *there is $m \in M$ such that the sectional curvature K of M at m satisfies $\frac{1}{4}A' < K \leq A'$ for some positive A'.*

Let $\langle , \rangle$ denote the metric tensor in R^{n+p}. Let X_i, $B(X_i, X_i)$, $2B(X_i, X_j) = 2B(X_j, X_i)$, $1 \leq i \neq j \leq n$, be unit vectors in R^{n+p} with the following properties:

(i) if $1 \leq i \neq j \leq n$, then $\{X_1, \cdots, X_n, B(X_i, X_i), 2B(X_i, X_j) = 2B(X_j, X_i)\}$ is orthonormal;

(ii) for every $i \neq j$, $1 \leq i, j \leq n$, $\langle B(X_i, X_i), B(X_j, X_j)\rangle = \frac{1}{2}$;

(iii) $\langle B(X_i, X_j), B(X_h, X_k)\rangle = 0$, for i, j, h differernt and $1 \leq i, j, h, k \leq n$.

Let c be a fixed positive real number, and m be a fixed point of R^{n+p}. By identifying points of R^{n+p} with their position vectors, the set of all points $\varphi(x_1, \cdots, x_n)$ defined by

Received January 7, 1972.