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A NEW METHOD FOR INFINITESIMAL RIGIDITY OF SURFACES WITH $K > 0$

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1. Introduction

Classical methods of proving infinitesimal rigidity of convex surfaces use an auxiliary vector function $\boldsymbol{y}$ and integral identities involving it. In this paper a new method will be given using $\boldsymbol{y}$. The main tool is the Lemma (3.0). The lemma applies directly to surfaces (bounded or not) with spherical image in a hemisphere. Where this assumption does not hold the well-known projective transformation of Darboux is used to obtain a surface and a deformation to which the lemma can be applied.

Infinitesimal rigidity is proved for various standard cases, including surfaces with fixed boundaries, Rembs boundaries (along which the unit surface normal is constant) and closed convex surfaces. If the spherical image does lie in a hemisphere, various results seem to be new, namely, Theorems (4.1), (4.2), and also (4.3), in which rigidity is proved while allowing the deformation (velocity) field to have *different* constant values on different boundary components. What also seems new is that these results hold even if the surface is not convex in the large, e.g., if the surface intersects itself or is many-sheeted.

The differentiability assumptions throughout will be (C'', C'') (that is the surface and the deformation respectively are both of class C'') as contrasted with (C''', C''') for classical methods, e.g., of Blaschke [1, p. 75], and with (C'', C') for the method of Minagawa and Rado [3] (the latter assume C''' for the surface in a neighborhood of a Rembs boundary).

The method has the unifying feature that for all the problems solved here the boundary conditions are expressed in the form $\boldsymbol{y}\cdot\boldsymbol{n} = \text{const.}$; cf. (4.1).

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2. General remarks

We state here definitions and hypotheses, which will hold throughout, as well as some general results of the theory of infinitesimal deformations. For a complete discussion see [1, Chap. VI].

$\boldsymbol{r} = \boldsymbol{r}(u, v)$ is the radius vector of a surface S (two-dimensional differentiable manifold) immersed in euclidean 3-space, joining the origin $(0, 0, 0)$ to a point