

A CUT POINT THEOREM FOR CAT(0) GROUPS

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Abstract

Let G be a group acting geometrically on a CAT(0) space X . We show that if $c \in \partial X$ is a cut point, then there is an infinite torsion subgroup of G which fixes c . In particular if G is virtually torsion free, if X is a Euclidean cube complex, or if X is 2-dimensional, then ∂X has no cut point.

We also show that if G is a group acting geometrically on a CAT(0) space X , then G has an element of infinite order.

Introduction

The purpose of this note is two-fold. First we will provide a proof of the cut point theorem for CAT(0) groups. (The notion of the boundary of a CAT(0) space will be defined in Section 2.)

Main Theorem. *Let G be a one-ended group acting properly discontinuously and cocompactly by isometries on a CAT(0) space X . If $c \in \partial X$ is a cut point, then there is an infinite torsion subgroup of G which fixes c .*

Secondly we will give a much shorter, largely self contained proof of the cut point theorem for word hyperbolic groups. The pieces of the original proof appear in [6], [7], [8], [9], [17], and [21].

Summary of Proof for both word hyperbolic and CAT(0)

We start with G , a group. In the case where G is word hyperbolic, let X be the Rips complex for G . In the CAT(0) setting, X will be a CAT(0) space admitting a geometric G -action. In either case G acts by

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