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DEHN SURGERY AND NEGATIVELY CURVED 3-MANIFOLDS

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1. Introduction

Dehn surgery is perhaps the most common way of constructing 3-manifolds, and yet there remain some profound mysteries about its behaviour. For example, it is still not known whether there exists a 3-manifold which can be obtained from S^3 by surgery along an infinite number of distinct knots.[1] (See Problem 3.6 (D) of Kirby's list [9]). In this paper, we offer a partial solution to this problem, and exhibit many new results about Dehn surgery. The methods we employ make use of well-known constructions of negatively curved metrics on certain 3-manifolds.

We use the following standard terminology. A *slope* on a torus is the isotopy class of an unoriented essential simple closed curve. If s is a slope on a torus boundary component of a 3-manifold X, then $X(s)$ is defined to be the 3-manifold obtained by Dehn filling along s. More generally, if $s_1, \dots, s_n$ is a collection of slopes on distinct toral components of ∂X, then we write $X(s_1, \dots, s_n)$ for the manifold obtained by Dehn filling along each of these slopes.

We also abuse terminology in the standard way by saying that a compact orientable 3-manifold X, with ∂X a (possibly empty) union of tori, is *hyperbolic* if its interior has a complete finite volume hyperbolic

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[1]Since this paper was written, John Osoinach has constructed a family of 3-manifolds, each with infinitely many knot surgery descriptions [Ph.D. Thesis, University of Texas at Austin].