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A LOCAL PROOF OF PETRI'S CONJECTURE AT THE GENERAL CURVE

HERB CLEMENS

Abstract

A proof of Petri's general conjecture on the unobstructedness of linear systems on a general curve is given, using only the local properties of the deformation space of the pair (curve, line bundle).

1. Introduction

Let L_0 denote a holomorphic line bundle of degree d over a compact Riemann surface C_0. The Petri conjecture stated that, if C_0 is a curve of general moduli, the mapping

$$\mu_0 : H^0(L_0) \otimes H^0\left(\omega_{C_0} \otimes L_0^{\vee}\right) \rightarrow H^0(\omega_{C_0})$$

is injective. Later, this assertion was given a more modern interpretation making it a central question in the study of curves and their linear series—what is now called Brill-Noether theory.

To recap the modern formulation we proceed as in [1]. Let $C_0^{(d)}$ denote the d-th symmetric product of C_0 and let $\Delta \subseteq C_0^{(d)} \times C_0$ denote the tautological divisor. Let

$$\mathbb{P}^r = \mathbb{P}\left(H^0(L_0)\right).$$

For the projection

$$p_* : C_0^{(d)} \times C_0 \rightarrow C_0^{(d)}$$

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