

THE MAGID-RYAN CONJECTURE FOR EQUIAFFINE HYPERSPHERES WITH CONSTANT SECTIONAL CURVATURE

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Abstract

We study affine hyperspheres with constant sectional curvature. More precisely we obtain a classification of the affine hyperspheres with constant sectional curvature c , provided $c \neq H$, where H denotes the affine mean curvature of the immersion. Our classification gives a complete and positive answer to a conjecture of M. Magid and P. Ryan about these hyperspheres.

1. Introduction

In this paper, we study nondegenerate affine hypersurfaces M^n in \mathbb{R}^{n+1} . It is well known that on such hypersurfaces there exists a canonical transversal vector field ξ called the affine normal vector field. If for all $p \in M$, $\xi(p)$ passes through a fixed point (resp. is parallel), M^n is called a proper affine sphere (resp. improper affine sphere).

The standard models of affine spheres are the quadrics. Unlike in Euclidean geometry, where the only umbilical submanifolds are the spheres and the linear subspaces, the class of all equiaffine spheres is simply too large to classify. Therefore, in order to better understand the geometry of affine spheres, it is necessary to impose an extra condition. This

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