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The Remainder Term in the Local Limit Theorem for Independent Random Variables

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Introduction

Let $\{X_k, k=1, 2, \cdots\}$ be a sequence of independent random variables with $EX_k=0$, $EX_k^2=\sigma_k^2<\infty$ $(\sigma_k \geqq 0)$ and with distribution function $F_k(x)$, and suppose that each of X_k has a bounded density function $p_k(x)$. Furthermore, we suppose that some of σ_k^2 are not zero, in particular, we assume $\sigma_1^2>0$ without loss of generality. Write $s_n^2=\sum_{k=1}^n \sigma_k^2$, $Z_n=s_n^{-1}\sum_{k=1}^n X_k$, $f_k(t)=Ee^{itX_k}$, $\bar{f}_n(t)=Ee^{itZ_n}$, $R_k(z)=\int_{|u|>z} u^2 dF_k(u)$ and $Q_k(z)=\left|\int_{|u|\leqq z} u^3 dF_k(u)\right|$. Moreover, let $\bar{p}_n(x)$ be the density function of Z_n and $\phi(x)$ be the standard normal density function.

Let us denote two classes of functions $g(z)$ defined for all z as follows:

$$G=\{g(z)\,|\,g(z) \text{ is even on } (-\infty, \infty) \text{ and positive on } (0, \infty), \text{ and further } z/g(z) \text{ is non-decreasing on } (0, \infty)\}$$

and

$$G_0=\{g(z)\,|\,g(z)\in G, \text{ and in addition, } z^\alpha/g(z) \text{ is non-decreasing on } (0, \infty) \text{ for some } \alpha \text{ with } 0<\alpha<1\} .$$

For $g(z)\in G$, write

$$\lambda_k(g)=\sup_{z>0} g(z)R_k(z) , \qquad \mu_k(g)=\sup_{z>0} \frac{g(z)}{z}Q_k(z) ,$$

$$\rho_k(g)=\lambda_k(g)+\mu_k(g)$$

and

$$T_n=\frac{\sum_{k=1}^n \rho_k(g)}{s_n^2 g(s_n)} .$$

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