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Invariants of Finite Abelian Groups Generated by Transvections

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Introduction

Let k be a field of characteristic p and G be a finite subgroup of $GL(V)$ where V is a k-space. Then G acts naturally on the symmetric algebra $k[V]$ of V. In the case of $p=0$ or $(|G|, p)=1$, it is well known (e.g., [1], [4]) that the invariant subring $k[V]^G$ is a polynomial ring if and only if G is generated by pseudo-reflections in $GL(V)$.

Suppose that $p \mid |G|$. We have classified in [2] finite irreducible groups G such that $k[V]^G$ are polynomial rings under certain conditions. In this paper we try to classify the modular representations of finite abelian groups with regular rings of invariants. Our main result is the following

THEOREM. *Let G be an abelian p-subgroup of $GL(V)$ which is realizable on F_p. If* $\dim V^G \leqq 2$ *or* $\dim V^{*G} \leqq 2$, *then the following conditions are equivalent*:

(1) *$k[V]^G$ is a polynomial ring.*

(2) *There exist couples $(W_i, G_i)(1 \leqq i \leqq m)$ which satisfy the following*

(i) *$G_i (1 \leqq i \leqq m)$ are subgroups of G and $G=\bigoplus_{i=1}^m G_i$.*

(ii) *$W_i (1 \leqq i \leqq m)$ are 1-dimensional subspaces of V and $V=V^G \oplus \bigoplus_{i=1}^m W_i$.*

(iii) *Each $V^G \oplus W_i$ is a kG_i-submodule $(1 \leqq i \leqq m)$ of V and $W_i \subseteqq V^{G_j}$ if $i \neq j$.*

Furthermore we will give some remarks concerned with singular loci of the rings $k[V]^G$ and examples of abelian p-groups generated by transvections with the invariant subrings which are not Macaulay rings. It should be noted that there are finite irreducible subgroups G of $GL(V)$

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