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Einstein Parallel Kaehler Submanifolds in a Complex Projective Space

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Introduction

Submanifolds with parallel second fundamental form (which are simply called parallel submanifolds) have been studied by many differential geometers. In particular, parallel Kaehler submanifolds in a complex projective space are completely determined (see [1]).

In this paper, we give some characterizations of Einstein parallel Kaehler submanifolds in a complex projective space.

Let $X: M \rightarrow E^N$ be an isometric immersion of an n-dimensional compact Riemannian manifold into an N-dimensional Euclidean space. We denote by Δ and $\mathrm{Spec}(M)=\{0<\lambda_1<\lambda_2<\cdots\}$, the Laplacian acting on differentiable functions of M and the spectrum of Δ, respectively. Then, X can be decomposed as $X=X_0+\sum_{k\in N} X_k$, where X_k is a k-th eigenfunction of Δ of M, X_0 is a constant mapping, and the addition is convergent componentwise for the L^2-topology on $C^\infty(M)$. We say that the immersion is of *order* $\{l\}$ (or mono-order) if $X=X_0+X_l$, $l\in \boldsymbol{N}$, $X_l\neq 0$, and of *order* $\{k, l\}$ (or bi-order) if $X=X_0+X_k+X_l$, $k, l\in \boldsymbol{N}$, $l>k$, $X_k, X_l\neq 0, \cdots$ (see [4]).

Let $F: \boldsymbol{C}P^m \rightarrow E^N$ be the first standard imbedding of an m-dimensional complex projective space of constant holomorphic sectional curvature 1 into an N-dimensional Euclidean space, and $i: M^n \rightarrow \boldsymbol{C}P^m$ be a Kaehler immersion of an n-dimensional compact Kaehler manifold. We consider $\phi=F\circ i: M^n \rightarrow E^N$. Then, ϕ is mono-order if and only if M is totally geodesic (See [3].), and totally geodesic Kaehler submanifolds are of order 1. Let A be the shape operator of the immersion i, and define the tensor T by

$$T(\xi, \eta)=\mathrm{tr}\, A_\xi A_\eta \qquad \text{for} \quad \xi, \eta\in NM\,,$$

where NM is the normal bundle of M. Then T is a symmetric bilinear mapping from $NM\times NM$ into $\boldsymbol{R}$. A. Ros [3] has proved that M is bi-order

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