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Ergodic Measure Preserving Transformations of Finite Type

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§1. Introduction.

In this paper we shall consider properties of ergodic measure preserving (e.m.p.) transformations T defined on an infinite (σ-finite) Lebesgue measure space $(X, \mathscr{B}, m)$. It is well known that, in general, properties of such transformations are quite different from those of e.m.p. transformations defined on a finite measure space. For example, if $m(X)<\infty$ then it is easy to show that any non-singular measurable transformation S defined on $(X, \mathscr{B}, m)$ satisfying $ST=TS$ must preserve the same measure m; this need not be the case if $m(X)=\infty$, see [8]. Furthermore, if $m(X)=\infty$, then the L^∞-point spectrum $\Lambda(T)$ can be uncountable; [1], [9], [12].

A distinguishing feature of e.m.p. transformations T defined on a σ-finite measure space is the fact that if $m(X)=\infty$ then T always admits weakly wandering (w.w.) sets of positive measure, and hence w.w. sequences; this is never the case if $m(X)<\infty$.

In [7], an example of an e.m.p. transformation T was constructed which possessed an exhaustive (exh.) w.w. sequence. Namely, an infinite sequence $\{n_i\}$ of integers for which there exists a measurable set W such that $T^{n_i}W\cap T^{n_j}W=\varnothing$ for $i\neq j$, and $\bigcup_i T^{n_i}W=X$. It was shown later in [10] that every e.m.p. transformation T defined on an infinite measure space admits an exh. w.w. sequence $\{n_i\}$. However, sets which are exh. and w.w. under such sequences may or may not be of finite measure. In [4] and [5] a class of e.m.p. transformations is constructed which admit exh. w.w. sequences $\{n_i\}$ for which the corresponding w.w. sets W must have finite measure; we designate these as transformations of finite type. We will show in Theorem 1 below that for any e.m.p. transformation T defined on an infinite measure space $(X, \mathscr{B}, m)$ sets which are exh. and w.w. for T under the same sequence $\{n_i\}$ must have the same measure, finite or infinite. Hence for a given e.m.p. transformation T defined on

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