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Harmonic Bloch and BMO Functions on the Unit Ball in Several Variables

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Dedicated to Professor Junzo Wada on his 60th birthday

Introduction.

Harmonic Bloch functions on the unit ball are those harmonic functions h for which the quantity $|\nabla h(x)|(1-|x|^2)$ is bounded (for x in the ball). We prove that the space of harmonic Bloch functions on the unit ball is isomorphic to the space of harmonic BMO functions on the unit ball as Banach spaces. In this proof, we use the stochastic theory to give a good estimate (inequality (1.2) in Theorem).

§1. Preliminaries and the main theorem.

Let D_n be the unit ball in the n dimensional Euclidian space $(n\geqq 2)$ and $H(D_n)$ the space of real harmonic functions on D_n.

A function h in $H(D_n)$ is said to be a *harmonic Bloch function* if

$$\|h\|_{H,n}=\sup_{x\in D_n}\frac{1}{2}(1-|x|^2)|\nabla h(x)|<\infty\ ,$$

where $|\nabla h(x)|=\{\sum_{i=1}^{n}(\partial h(x)/\partial x_i)^2\}^{1/2}$. The space of harmonic Bloch functions is denoted by $B_H(D_n)$.

Let $p\geqq 1$. A function h in $H(D_n)$ is said to be a *harmonic* BMO_p *function* if

$$\|h\|_{p,n}=\sup\left\{\frac{1}{|B|}\int_B |h(x)-h(b)|^p dx\right\}^{1/p}<\infty$$

where the supremum is taken over all balls B in D_n, $|B|=\int_B dx$, and b is

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