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Smooth Perturbations of the Self-adjoint Operator $|\Delta|^{\alpha/2}$

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1. Introduction.

In this paper we shall consider smooth perturbations of the formal self-adjoint operator $|\Delta|^{\alpha/2}$ in $L^2(\boldsymbol{R}^n)$, where $\Delta=\sum_{i=1}^m \partial^2/\partial x_i{}^2$. We shall first recall some notations in the theory of smooth perturbations.

Let H be a selfadjoint operator in a separable Hilbert space $\boldsymbol{H}$ with its resolvent denoted by $R(\zeta)=(H-\zeta)^{-1}$, $\operatorname{Im}\zeta\neq 0$. A densely defined closed linear operator A is said to be smooth with respect to H, H-smooth for short, if

$$\int_{-\infty}^{+\infty}\|AR(\lambda\pm i\varepsilon)u\|^2 d\lambda\le c_1^2\|u\|^2\,,\qquad u\in\boldsymbol{H}\,,\quad \varepsilon>0\,, \tag{1.1}$$

where c_1 is a constant independent of u and $\varepsilon>0$. Each of the following conditions (1.2) and (1.3) is equivalent to (1.1) (cf. T. Kato [2]):

$$|\operatorname{Im}(R(\zeta)A^*u, A^*u)|\le c_2^2\|u\|^2\,,\qquad u\in D(A^*)\,,\quad \operatorname{Im}\zeta\neq 0\,; \tag{1.2}$$

$$\int_{-\infty}^{+\infty}\|Ae^{-itH}u\|^2 dt\le c_3^2\|u\|^2\,,\qquad u\in\boldsymbol{H}\,. \tag{1.3}$$

Here $c_2>0$ and $c_3>0$ are constants independent of u and ζ. $\{e^{itH}\}_{t\in\boldsymbol{R}}$ is a unitary group generated by H, and it is understood that $\|Ae^{-itH}u\|=\infty$ if $e^{-itH}u\notin D(A)$. For more details, see T. Kato [2]. A is said to be supersmooth with respect to H, H-supersmooth for short, if

$$|(R(\zeta)A^*u, A^*u)|\le c_4^2\|u\|^2\,,\qquad u\in D(A^*)\,,\quad \operatorname{Im}\zeta\neq 0\,, \tag{1.4}$$

where c_4 is a constant independent of u and $\operatorname{Im}\zeta\neq 0$. This terminology was introduced by T. Kato and K. Yajima [3], but the notion itself appeared in T. Kato [2].

T. Kato and K. Yajima proved in [2] that $A=|x|^{-\beta}|\nabla|^{1-\beta}$ with $1/2<\beta\le 1$ is $-\Delta$-supersmooth. The $-\Delta$-smoothness was also proved by other simple methods (see M. Ben-Artzi and S. Klainerman [1], B. Simon [8]) and was extended to the Schrödinger

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