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Moves for Flow-Spines and Topological Invariants of 3-Manifolds

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Introduction.

A spine P for a closed 3-manifold M is a 2-dimensional polyhedron in M such that the complement of the regular neighborhood of P is homeomorphic to the 3-ball. Cutting off a closed 3-manifold M along its spine P, we get a 3-ball B^3 with an identification on its boundary. This is a polyhedral representation of M, which is first considered by M. Dehn in the case of closed surfaces, and introduced by H. Seifert in the 3-dimensional case.

A DS-diagram is a polyhedral representation of a special class, which was first introduced in [3]. A spine corresponding to a DS-diagram forms a closed fake surface (cf. [1], [3]). A spine which forms a closed fake surface is called a standard or a simple spine. As is pointed out in [12], a standard spine is the dual of a singular triangulation.

A flow-spine introduced in [7] is a standard spine of a more special class, which is generated by a pair of a non-singular flow and its local section. It was shown in [4] and [7] that a DS-diagram for a flow-spine has an E-cycle. An E-cycle is a cycle of the graph of a DS-diagram which represents a kind of symmetry of a polyhedral representation. (See §1 for precise.)

A closed 3-manifold admits infinitely many flow-spines. In this paper, we shall give conditions for two flow-spines to represent the same manifold, that is, it will be shown that any two flow-spines of a 3-manifold can be transformed from one to another by a finite sequence of operations of three types which we call "moves". A flow-spine is completely determined by a data on the E-cycle, which will be called an E-data (cf. §1). An E-data is the one called a singularity-data in [7]. Our moves of flow-spines are described in terms of E-data. For an easy description of moves of E-data, we introduce the graphic representation of an E-data in §1.

An E-data determines not only a 3-manifold M but also a class of non-singular flows on M (see §1). Moves of E-data are divided into two types, moves which do not change the class of non-singular flows and those which change the class. Moves of the first type are called regular moves and discussed in §2. The second type consists of only

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