TOKYO J. MATH.
VOL. 16, NO. 1, 1993

Remarks on Pitman Deficiency

Haruyoshi MITA

University of the Sacred Heart

(Communicated by Y. Shimizu)

Introduction.

Let the distributions P_θ be indexed by parameter θ in a set Θ, where Θ is a subset of $\boldsymbol{R}^1$. We consider the testing problem

$$H:\theta=\theta_0 \qquad \text{against} \quad K:\theta>\theta_0\,.$$

In case that the alternative is close to the null hypothesis, we attempt to compare two tests. A method of the comparison of two tests in the local sense was given by Pitman (Noether [5], Pitman [6]). Pitman introduced the concept of asymptotic relative efficiency of two tests by choosing alternative sequences that approach to the null hypothesis. Roughly speaking, his method is as follows. Let $\{T_{1n_1}\}$, $\{T_{2n_2}\}$ be two tests based on n_1, n_2 samples, respectively, and α_{in_i}, $\beta_{in_i}(\theta)$ $(i=1,2)$ denote the corresponding levels and power functions. For $i=1,2$ suppose that $\alpha_{in_i}\to\alpha$ $(0<\alpha<1)$ as $n_i\to\infty$, and choose the alternative sequence $\{\theta_{in_i}\}$ approaching to the null hypothesis θ_0 so that $\beta_{in_i}(\theta_{in_i})\to\beta$ $(0<\beta<1)$ as $n_i\to\infty$. Then Pitman defined the asymptotic relative efficiency (ARE) of $\{T_{2n}\}$ with respect to $\{T_{1n}\}$ as the limit of the ratio n_1/n_2. The superiority or inferiority between $\{T_{1n}\}$ and $\{T_{2n}\}$ in the local sense is decided whether ARE>1 or ARE<1. If ARE$=1$ then we consider the limit of the difference of sample sizes n_2-n_1, what is called Pitman deficiency, as the second measure of comparison of the two tests. In many cases it occurs that $\theta_{in}=\theta_0+k_i/\sqrt{n}$. But this alternative form is not appropriate for the study of deficiency, because approaching to the null hypothesis is coarse. And so we choose the alternative sequence of the form $\theta_{in}=\theta_0+k_i/\sqrt{n}+l_i/n+m_i/(n\sqrt{n})$ $(i=1,2)$. By expanding the power functions we compare the two tests under these alternative sequences. Here l_i may be related to the case when Pitman deficiency is infinite. In this paper, however, we study the case when Pitman deficiency is finite only.

In section 1 we consider a method of comparison of two tests in such a case that ARE$=1$. In section 2 the method is applied to two examples and in section 3 we refer to the relation between our method and Pitman deficiency.

Received April 6, 1992