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Nonexistence Results for Harmonic Maps between Noncompact Complete Riemannian Manifolds

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§1. Introduction.

In this paper, we prove two kinds of nonexistence results for harmonic maps. The one is to prove nonexistence of a harmonic map with a rotational nondegeneracy at infinity, from a simple Riemannian manifold to an Hadamard manifold of negative sectional curvature bounded away from zero. The other is to prove nonexistence of a nonconstant harmonic map with a polynomial growth dilatation, from a complete Riemannian manifold of nonnegative Ricci curvature to a Riemannian manifold of negative sectional curvature bounded away from zero.

Let $M=(M^m, h)$ and $N=(N^n, g)$ be Riemannian manifolds of dimension m and n $(m, n\geq 2)$ respectively. Throughout this paper we denote by $x=(x^1, \cdots, x^m)$ and $y=(y^1, \cdots, y^n)$ local coordinates on M and N respectively. We shall write $(h_{\alpha\beta}(x))$ and $(g_{ij}(y))$ for the metric tensors with respect to the local coordinates on M and N respectively. Moreover, $(h^{\alpha\beta}(x))=(h_{\alpha\beta}(x))^{-1}$, $(g^{ij}(y))=(g_{ij}(y))^{-1}$ and $h(x)$ denotes the determinant of $(h_{\alpha\beta})$. The Christoffel symbols on M and N will be denoted by $\Gamma^{\alpha}_{\beta\gamma}$ and Γ^{i}_{jk} respectively.

For a map $U\in C^1(M, N)$ we define the *energy density* $e(U)(x)$ of U at $x\in M$ by

$$e(U)(x)=\frac{1}{2}\|dU(x)\|^2=\frac{1}{2}h^{\alpha\beta}(x)D_\alpha u^i(x)D_\beta u^i(x)g_{ij}(u(x)),$$

where $u(x)$ is the expression of $U(x)$ with respect to the local coordinates $(y^1, \cdots, y^n)$ and D_α denotes $\partial/\partial x^\alpha$. For a bounded domain $\Omega\subset M$, we define the *energy* of U on Ω by

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