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On Helicoidal Surfaces with Constant Mean Curvature and Their Limiting Surfaces

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Dedicated to Professor Kenjiro Okubo on his 60th birthday

Introduction.

In this paper we shall study the family of helicoidal surfaces $f: M^2 \to \mathbf{E}^3$ with non-zero constant mean curvature in the Euclidean 3-space $\mathbf{E}^3$, which was investigated by M. P. doCarmo and M. Dajczer [doC-D]. They obtained an integral representation of f which contains three parameters, say B, h and m (see Theorem 1.1 below). It gives a natural generalization of a representation for Delaunay surfaces, i.e., the surfaces of revolution with constant mean curvature, due to K. Kenmotsu [K].

Furthermore, for a given helicoidal surface $f: M^2 \to \mathbf{E}^3$, they determined a 2π-periodic parametrization of a family of immersions $f_\theta: M^2 \to \mathbf{E}^3$, $\theta \in [0, 2\pi]$, $f_0 = f$, all of which are isometric to f and have the same constant mean curvature H (see Theorem 1.3). The family $\{f_\theta\}$ is called the *associated family* to f and contains, except for a few cases, just two different Delaunay surfaces. Lawson's theorem ([L], see also [doC-D], p. 426) says that the family $\{f_\theta\}$ contains all local isometric immersions with given H. In §1 we shall summarize several known results for the later use.

Their representation of the family $\{f_\theta\}$ emphasizes the Delaunay surfaces as the starting point of deformations. However, for an arbitrarily given helicoidal surface, it is slightly hard to find the family to which it belongs. So, in §2, we shall construct a new parametrization $S(B, \mathfrak{h}, \mathfrak{m})$ of the family which also depends on one parameter B. It takes a given helicoidal surface as the starting point of its deformations from which the corresponding values of the remaining parameters h and m are calculated at once. If a given helicoidal surface f is not the right circular cylinder, then, in the both of these parametrizations f_θ and $S(B, \mathfrak{h}, \mathfrak{m})$, the parameter B moves on the closed interval bounded by B^+ and $1/B^+$ (see (A. 1) in §2), which is determined uniquely by f. However, it seems unnatural that, in the family $\{f_\theta\}$ for the right circular cylinder corresponding to $B=0$, the parameter B stays at zero only. Moreover, there appears only one limiting

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