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On the Generalized Thomas-Fermi Differential Equations and Applicability of Saito's Transformation

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Dedicated to Professor Junji Kato on his sixtieth birthday

1. Introduction.

Let us consider the generalized Thomas-Fermi differential equation

$$x''=P(t)x^{1+\alpha}, \qquad '=d/dt, \quad x\geqq 0 \tag{1.1}$$

where α is a nonzero real constant and $x^{1+\alpha}$ denotes a nonnegative-valued branch.

In the papers [5], [6] Saito succeeded in investigating the asymptotic behavior of solutions of (1.1) where $P(t)=t^{\alpha\lambda-2}$ (λ is a positive constant) with the aid of a transformation

$$y=\psi(t)^{-\alpha}\phi(t)^{\alpha}, \qquad z=ty' \tag{1.2}$$

which transforms (1.1) to a first order algebraic differential equation

$$\frac{dz}{dy}=\frac{-\lambda(\lambda+1)\alpha^2y^2+(2\lambda+1)\alpha yz-(1-\alpha)z^2+\lambda(\lambda+1)\alpha^2y^3}{\alpha yz}. \tag{1.3}$$

In (1.2), $\psi(t)=[\lambda(\lambda+1)]^{1/\alpha}t^{-\lambda}$ is a particular solution of (1.1) and $\phi(t)$ is an arbitrary solution of (1.1). Moreover in [8], [9] we considered the case $P(t)=\pm e^{\alpha\lambda t}$ where λ is a real constant, using a transformation in a form similar to (1.2) such as

$$y=\psi(t)^{-\alpha}\phi(t)^{\alpha}, \qquad z=y'$$

where $\psi(t)=\pm\lambda^{2/\alpha}e^{-\lambda t}$. This transforms (1.1) to a first order algebraic differential equation also.

Since the coefficients of y' in the two transformations above differ, we consider a more general transformation

$$y=\psi(t)^{-\alpha}\phi(t)^{\alpha}, \qquad z=\theta(t)y' \tag{1.4}$$

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